Let X and Y denote two random variables. Then the conditional variance of Y given $X = x$ is

$$Var(Y|X = x) = \int \left(y - \left(\int y f_Y(z) dz \right) \right)^2 f_{Y|X}(y|x) dy$$

where $f_Y(y)$ is the marginal density of Y and $f_{Y|X}(y|X)$ the conditional density of Y given X. Is this expression correct or incorrect?

Question 1

(a) correct

(b) incorrect

Let $X_1, \ldots, X_n$ be i.i.d. random variables with positive mean μ and finite, positive variance σ^2. Then

$$\frac{\sum_{i=1}^{n} X_i}{\sum_{i=1}^{n} X_i^2}$$

is a consistent estimator of

Question 2

(a) μ/σ^2

(b) μ/σ

(c) $\mu/(\sigma^2 + \mu^2)$

(d) $1/\mu$

Let $X_1, \ldots, X_n$ be an i.i.d. sequence of random variables with mean 0 and variance 2. Also, denote $\bar{X}_n = \frac{1}{n}\sum_{i=1}^{n} X_i$. Then, what is the limiting distribution of

$$\frac{\sqrt{n}\bar{X}_n - 1}{\frac{1}{n}\sum_{i=1}^{n} X_i^2 - 1}?$$

Question 3

(a) $N(0, 2)$

(b) $N(-1, 2)$

(c) $N(1, 2)$

(d) $N(0, 1)$

Let $X_1, \ldots, X_n, W_1, \ldots, W_n$ be i.i.d. random variables that are normally distributed. Let $Y_i = X_i + W_i$, denote by $\bar{X}_n$, $\bar{W}_n$ the sample means of X_i, W_i, and let $S^2_{X,n}$, $S^2_{W,n}$ be the sample variances of X_i, W_i, respectively. Which of the following is a valid 95% confidence interval for the mean of Y_i?

where z is the 0.975-quantile of the standard normal distribution.

Question 4

(a) $\left[\bar{X}_n + \bar{W}_n - z\frac{S_{X,n}+S_{W,n}}{\sqrt{n}},\ \bar{X}_n + \bar{W}_n + z\frac{S_{X,n}+S_{W,n}}{\sqrt{n}}\right]$

(b) $\left[\bar{X}_n + \bar{W}_n - z\frac{\sqrt{S^2_{X,n}+S^2_{W,n}}}{\sqrt{n}},\ \bar{X}_n + \bar{W}_n + z\frac{\sqrt{S^2_{X,n}+S^2_{W,n}}}{\sqrt{n}}\right]$

(c) $\left[\bar{X}_n + \bar{W}_n - z\frac{S^2_{X,n}+S^2_{W,n}}{n},\ \bar{X}_n + \bar{W}_n + z\frac{S^2_{X,n}+S^2_{W,n}}{n}\right]$

(d) $\left[\bar{X}_n - z\frac{S_{X,n}}{\sqrt{n}},\ \bar{X}_n + z\frac{S_{X,n}}{\sqrt{n}}\right]$

5. Suppose the two random variables X and Y each take values in $\{0, 1\}$. Assume $P(X = 0, Y = 0) = 0.25$, $P(X = 1, Y = 1) = 0.1$, and $P(Y = 0) = 0.5$. Then, what is $P(Y = 1|X = 1)$?

question 5

(a) $\frac{1}{2}$

(b) $\frac{4}{5}$

(c) $\frac{1}{5}$

(d) $\frac{2}{7}$

6. Let X be a random variable that is uniformly distributed over the interval $[0, 1]$, i.e.

$$f_X(x) = \begin{cases} 1, & 0 \le x \le 1 \\ 0, & otherwise \end{cases}.$$

What is the density, $f_Y(y)$, of $Y = \frac{1}{3}X^2$?

question 6

(a) $f_Y(y) = \begin{cases} \frac{1}{3}, & 0 \le y \le 3 \\ 0, & otherwise \end{cases}$

(b) $f_Y(y) = \begin{cases} \frac{1}{2\sqrt{y}}, & 0 \le y \le 1 \\ 0, & otherwise \end{cases}$

(c) $f_Y(y) = \begin{cases} \frac{3}{2\sqrt{3y}}, & 0 \le y \le 1/3 \\ 0, & otherwise \end{cases}$

(d) $f_Y(y) = \begin{cases} \frac{1}{2\sqrt{3}}, & 0 \le y \le 1 \\ 0 & otherwise \end{cases}$

7. Suppose two random variables X and Y have the joint density

$$f_{Y,X}(y,x) = \begin{cases} 2e^{-2x-y}, & x \geq 0, y \geq 0 \\ 0, & otherwise \end{cases}.$$

Then:

question 7

(a) X and Y are independent.

(b) X and Y are not independent.

8. Let Y, X, U be random variables such that $Y = X + U$, X and U take values in $\{0, 1\}$, and X and U are independent. Suppose $P(U = 1) = \frac{1}{3}$. Then what is $E(Y|X = 1)$?

question 8

(a) $\frac{1}{12}$

(b) 1

(c) $\frac{7}{12}$

(d) $\frac{4}{3}$